Kaneko and Toyama (2025)

Student Presentation in Empirical Industrial Organization

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- A semiparametric discrete choice model
  - Proposing nonparametric sieve approximation of income effect
  - Resulting in more accurate estimation of demand curvature, price elasiticity, and welfare changes.
- Empirical application[1]
  - A feebate policy in the Japanese automobile industry[2]
  - High pass-through rate
  - More significant merger effects (Toyota & Honda)

[1] To be skipped in the class.
[2] Subsidy for eco-friendly cars.

Introduction

Consumer Demand

- Accurate measurement of consumer demand is critical.
    - Price elasticity and substitution patterns are often what firms must consider.
    - Decision-making on pricing in oligopolistic markets
    - Evaluating the welfare consequences

Specification for Income Effects

- When estimating consumer demand for differentiated products, it is common to rely on parametric specifications.
- However, such parametrizing often imposes strict restrictions on the shape of demand curve.
- A semiparametric discrete choice model can adress this concern:
- This allows for the flexible estimation of demand curvature and price elasticity patterns.

Demand Estimation with Flexible Income Effect

- Combine a method of sieve approximation (Chen, 2007) and nested fixed point algorithm (BLP):
- First, approximate the income effect by nonparametric sieve methods.
- Then, their model is closely aligns with the standard parametric framework of BLP.
- Second, implement a nested fixed-point algorithm to run sieve GMM estimation.

Demand Model

Utility Maximization Problem

- Let $U(m, j)$ denote the direct utility function.
  - m is a d_m dimensional vector representing the consumption of continuous choice goods.
  - $j \in \mathbb{J} = \{0, 1, \cdots, J\}$ corresponds to an alternative in the discrete choice decision, with J products available in the market. The index $j = 0$ indicates the outside goods.
- The utility maximization problem is given by

$$\max_{(m,j)\in\mathbb{R}_+^{d_m}\times\mathbb{J}} \quad U(m, j) \qquad (1)$$
$$\text{s.t.} \quad P_m^T m + p_j \le y_i,$$

where P_m is a d_m dimensional vector of prices of continuous choice goods, p_j is the price of alternative j, and y_i is income.

Conditional Indirect Utility Function

- Conditional on choice j in the discrete choice, the conditional indirect utility function is defined as

$$V(P_m, y - p_j, j) \equiv \max_{m \in \mathbb{R}_+^{dm}} U(m, j) \text{ s.t.} P_m^T m \leq y_i - p_j. \quad (2)$$

Note that we define $p_0 = 0$ as choosing the outside good incurs no costs.

- Assume that the direct utility function satisfies

$$U(m, j) = v(j) + u(m). \quad (3)$$

- The conditional indirect utility function can be rewritten as

$$V(P_m, y - p_j, j) = v(j) + \tilde{V}(P_m, y - p_j). \quad (4)$$

Income Effect

- Assume that the continuous good is a numeraire, with its price represented by P^m. Then, we obtain

$$\tilde{V}(P^m, y - p_j) = u\left(\frac{y - p_j}{P^m}\right),$$

  implying that the utility from numeraire depends on the disposal income $y - p_j$ after choosing alternative j.

- Define the income effect term by

$$f(y - p_j) \equiv \tilde{V}(P^m, y - p_j).$$

  Note that $f(y - p_j)$ should be weakly-increasing.

Conditional Indirect Utility Function

- Letting v_{ij} denote consumer i's utility from a discrete choice good j, we specify that

$$v_{ij} = \beta^T X_j + \xi_j + \epsilon_{ij} \text{ for } j = 1, \cdots J, \tag{5}$$

$$v_{i0} = \epsilon_{i0}. \tag{6}$$

  where X_j is a vector of observable characteristics of product j, ξ_j represents its unobservable characteristics, and ϵ_{ij} is an IID idiosyncratic shock that follows the type I extreme-value distribution.

- Hence, the conditonal indirect utility function of consumer i when choosing j is given by

$$V_{ij} = \begin{cases} f(y_i - p_j) + \beta^T X_j + \xi_j + \epsilon_{ij} & \text{for } j = 1, \cdots J, \\ f(y_i) + \epsilon_{i0} & \text{for } j = 0. \end{cases} \tag{7}$$

Individual Choice Probability

- Define the choice set of consumer i as

$$\mathbb{J}_{it} = \{0\} \cup \{j \in \{1, \cdots, J_t\} : y_{it} - p_{jt} \geq 0\}, \qquad (8)$$

where J_t is the total number of products available in market t.

- Given the conditional indirect utility V_{ijt} (7), the discrete choice problem is described as

$$\max_{j \in \mathbb{J}_{it}} V_{ijt}. \qquad (9)$$

and the choice probability for consumer i selecting alternative j is derived as

$$s_{ijt}(y_{it}) = \frac{1(y_{it} \geq p_{jt}) \cdot \exp\left(f(y_{it} - p_{jt}) + \beta^T X_{jt} + \xi_{it}\right)}{\exp\left(f(y_{it})\right) + \sum_{k=1}^{J_t} 1(y_{it} \geq p_{kt}) \cdot \exp\left(f(y_{it} - p_{kt}) + \beta^T X_{kt} + \xi_{it}\right)}. \qquad (10)$$

Market Share

- Letting y_{it} follow the distribution of income $G_t(y_{it})$, the market share is given by

$$s_{jt} = \int s_{ijt} dG_t(y_{it}). \tag{11}$$

- Market demand q_{jt} is given by

$$g_{jt} = N_t \times s_{jt}$$

where N_t denote the market size.

Practical Importance of the Flexible Income Effect

- Price Elasiticity: Avoid imposing any predetermined restrictions on how own-price elasticity varies with price.
- Pass-Through Analysis: Avoid inherent restriction on the demand curvature.
- Merger Analysis: Different curvatures of the demand funtion lead different simulated merger outcomes even under the same consumer demand with identical elasticities.

Estimation Method

Estimation

- The utility function includes the nonparametric function $f(y-p)$ and the linear parameter β.
- Employ a sieve approximation for the nonparametric function and incorporate it into the nested fixed-point (NFP) algorithm.
- See Chen (2007) for sieve approximation, and BLP (1995) for NFP algorithm.

- Approximate $f(\cdot)$ by the K-th order Bernstein polynomial, i.e., by a linear function of the basis function $\Psi^K(x) = (b_0^K(x), b_1^K(x), \cdots b_K^K(x))^T$ and coefficients $\Pi = (\pi_0, \pi_1, \cdots \pi_k)^T$:

$$f(x) \simeq B_K(x) = \sum_{k=0}^{K} \pi_k b_k^K(x) \equiv \Psi^K(x)^T \Pi \tag{12}$$

where

$$b_k^K(x) = \binom{K}{k} x^k (1-x)^{K-k}, \tag{13}$$

and letting x be normalized to $[0, 1]$.

Shape Restrictions & Normalization

- Select the Bernstein polynomial as a basis function.
- Recall that $f(y-p)$ is weakly increasing (monotonicity). To incorporate this restriction within estimation, we impose constraints on the coefficients Π.
- Under $\pi_k \leq \pi_{k+1}$ for all k, the derivative of $B_K(x)$ (12) satisfies that

$$B'_K(x) = K \sum_{k=0}^{K-1} (\pi_{k+1} - \pi_k) b_k^{K-1}(x) \geq 0$$

  for all k, which is the desired monotonicity.
- The level of the income effect cannot be identified. Thus, letting $\pi_0 = 0$, we normalize $f(x)$ as $f(0) = 0$.

Approximated Model

- Under the sieve approximation above, the market share defined by (10) and (11) can be rewritten as

$$s_{jt} = \int \frac{1(y_{it} \geq p_{jt}) \cdot \exp\left(\Psi^K(y_{it} - p_{jt})^T \Pi + \beta^T X_j t + \xi_{jt}\right)}{\text{denom.}} dG_t(y_{it}), \tag{14}$$

where the denominator is given by

$$\exp\left(\Psi^K(y_{it})^T \Pi\right) + \sum_{k=1}^{J_t} 1(y_{it} \geq p_{jt}) \cdot \exp\left(\Psi^K(y_{it} - p_{jt})^T \Pi + \beta^T X_{kt} + \xi_{jt}\right)$$

- Note that there emerges an endogeneity between the product proce p_{jt} and the unobserved product characteristics ξ_{jt}.
- Introduce IVs, for example, proposed by BLP, Konishi & Zhao (2017), and Kitano (2022), among others.

Sieve GMM

- Moment Conditions: for $b = 1, \cdots, B$,

$$\mathbb{E}\left[\xi_{jt}(\theta)p_b(X_{jt}, W_{jt})\right] = 0, \tag{15}$$

where X_{jt} is a vector of exogenous variables, W_{jt} is a vector of IVs, $\theta = (\beta, \Pi)$, $\{p_b(X_{jt}, W_{jt})\}_{b=1,\cdots,B}$ is a sequence of known functions that can approximate any real-valued square-integrable functions of X_{jt} and W_{jt} as $B \to \infty$.

- GMM Criterion:

$$\xi(\theta)^T \tilde{P}\left(\tilde{P}^T \tilde{P}\right)^{-} \tilde{P}^T \xi(\theta)^T, \tag{16}$$

where $\xi(\theta)^T$ is a vector that stacks ξ_{jt}'s. The matrix $\tilde{P} = [P, P \otimes X]$ denotes a matrix of instruments, for the choice of which we follow Chetverikov et al. (2018).

NFP Algorithm

- Caluculation of the objective function & numerical optimization procedures are as follows:[3]
  - 1. Caluculate the vector of mean utility δ by applying a contraction-mapping algorithm.
  - 2. Run a linear regression of δ on X and obtain $\hat{\beta}$ and the residual $\hat{\xi}_{jt}$.
  - 3. Caluculate the value of the objective function (16).
  - 4. Run a nonlinear optimization routine over Π. [4]
- Inference: Generalized residual bootstrap (Chen & Pouzo, 2015).

[3] See BLP (1995) for details.

[4] Note that β appearing in the mean utility function can be obtained by employing a linear GMM (concentration out: Nevo, 2001).

Monte Carlo & Empirical Example

See Sections IV, V, VI, and VII of Kaneko & Toyama (2025).

Conclusion

Conclusion

- A new framework for a differentiated product demand model with a nonparametric income effect
- Estimate the semiparametric model with endogeneity by combining the NFP algorithm and a sieve approximation.
- Monte Carlo simulations suggest significant gains in estimating the nonparametric term of the income effect by incorporating the shape restriction (Skipped in the class).
- Applying their framework to Japanese automobile data, they demonstrate the importance of a flexible income effect specification (Skipped in the class).