

Regression Discontinuity Designs

Hansen (2022, Chapter 21)

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<https://yasu0704xx.github.io>

- **Regression discontinuity designs (RDDs)** are quasi-experimental designs which allow researchers to identify the causal effect of endogenous treatment on an outcome based on discontinuous policy rules.
- **Local randomization** is a key idea.
 - Consider a certain discontinuous rule under which treatment (e.g. college scholarship) is determined by whether a continuous covariate (e.g. admission score) is greater than a known threshold.
 - If all factors determined prior to the treatment are balanced just above and just below the threshold, the average causal effect can be estimated by comparing the mean outcome just above the threshold with that just below the threshold.

- Here we review Chapter 21 of Hansen (2022) [27].
- Excellent reviews/textbooks on regression discontinuity designs include Cattaneo and Titiunik (2022) [16], and Cattaneo, Idrobo and Titiunik (2021, 2024) [13] [14].
- The common software package is `rdrobust` by Calonico, Cattaneo, Farrell and Titiunik.
- 日本語の文献：
 - 川口・澤田 (2024) [53]
 - 末石 (2024) [55]
 - 高野 (2025) [54]

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Identification

Setup: Rubin Causal Model

- $Y_i(d) \in \mathbb{R}$, $d \in \{0, 1\}$: potential outcome
- Y_i : observed outcome
- $D_i \in \{0, 1\}$: treatment, which may be endogenous in that some unobserved factors may affect both D_i and Y_i .
- $\theta = Y_i(1) - Y_i(0)$: treatment effect for an individual
 \Rightarrow We cannot identify θ without restrictive assumptions, because either $Y_i(1)$ or $Y_i(0)$ is unobservable.
- Instead, we are often interested in causal parameters such as
 $ATE = \mathbb{E}[Y_i(1) - Y_i(0)]$, $ATT = \mathbb{E}[Y_i(1) - Y_i(0) | D = 1]$, etc.

- Suppose that treatment is determined by

$$D_i = 1(X_i \geq c), \quad (1)$$

where the cut-off c is determined by policy or rule and common to all individuals.

- The covariate X_i is called the score, forcing variable, running variable, assignment variable, etc.
- In a standard RD setting, X_i is assumed to be continuously distributed on a subset of \mathbb{R} .

Identification in Sharp RD

Assume that, for each $d \in \{0, 1\}$,

$$\mathbb{E}[Y_i(d)|X_i = x] \text{ is continuous at } x = c. \quad (2)$$

Under Assumptions (1) and (2), the average causal effect at the cutoff point $\tau_{SRD} \equiv \mathbb{E}[Y_i(1) - Y_i(0)|X_i = c]$ is identified by

$$\tau_{SRD} = \lim_{x \downarrow c} \mathbb{E}[Y_i|X_i = x] - \lim_{x \uparrow c} \mathbb{E}[Y_i|X_i = x], \quad (3)$$

where $\lim_{x \downarrow c}$ and $\lim_{x \uparrow c}$ denote the right and left limits at $x = c$, respectively.

Proof By construction,

$$Y_i = Y_i(0) \cdot 1(X_i < c) + Y_i(1) \cdot 1(X_i \geq c).$$

Taking expectations conditional on $X_i = x$, we obtain

$$\begin{aligned}\mathbb{E}[Y_i|X_i = x] \\ = \mathbb{E}[Y_i(0)|X_i = x]1(X_i < c) + \mathbb{E}[Y_i(1)|X_i = x]1(X_i \geq c)\end{aligned}$$

Since $\mathbb{E}[Y_i(0)|X_i = x]$ and $\mathbb{E}[Y_i(1)|X_i = x]$ are continuous at $x = c$, they are identified by

$$\begin{aligned}\mathbb{E}[Y_i(0)|X_i = c] &= \lim_{x \uparrow c} \mathbb{E}[Y_i|X_i = x], \\ \mathbb{E}[Y_i(1)|X_i = c] &= \lim_{x \downarrow c} \mathbb{E}[Y_i|X_i = x],\end{aligned}$$

which completes the proof.

- Assumption (2) means that the conditional expectation of the untreated and treated outcome are continuously affected by the running variable.
- It is implied that the distributions of confounding factors, including observable covariates determined prior to the treatment, are balanced near the cutoff.
- In particular, there should be no policy/legal/experimental changes at the cutoff, except for the treatment assignment.
- **Counterfactual:** The continuity of $\mathbb{E}[Y_i(d)|X_i = x]$ at $x = c$ cannot be directly examined, since $Y_i(0)$ and $Y_i(1)$ are unobservable under $X_i \geq c$ and $X_i < c$, respectively.

- In the fuzzy RD, D_i is partially determined by whether X_i is no less than a known fixed cutoff c , such that

$$\lim_{x \downarrow c} \mathbb{E}[D_i | X = x] \neq \lim_{x \uparrow c} \mathbb{E}[D_i | X = x],$$

where $\lim_{x \downarrow c}$ and $\lim_{x \uparrow c}$ denote the right and left limits at $x = c$, respectively.

- Notice that $\mathbb{E}[D_i | X_i] = \mathbb{P}(D_i = 1 | X_i)$.

Identification in Fuzzy RD

- Define $Z_i = 1(X_i \geq c)$.
- Let $D_i(z), z \in \{0, 1\}$ be the potential treatment status when $Z_i = z$. By construction, $D = Z_i D_i(1) + (1 - Z_i) D_i(0)$.
- Consider the following causal parameter for the “compliers:”

$$\tau_{\text{FRD}} \equiv \mathbb{E}[Y_i(1) - Y_i(0) \mid D_i(1) > D_i(0), X_i \in \{c - \epsilon, c + \epsilon\}].$$

- Under several assumptions, τ_{FRD} can be identified by the local Wald estimand:

$$\tau_{\text{FRD}} = \frac{\lim_{x \downarrow c} \mathbb{E}[Y_i | X_i = x] - \lim_{x \uparrow c} \mathbb{E}[Y_i | X_i = x]}{\lim_{x \downarrow c} \mathbb{E}[D_i | X_i = x] - \lim_{x \uparrow c} \mathbb{E}[D_i | X_i = x]}. \quad (4)$$

- The arguments are quite similar to the identification of LATE parameter in the IV estimations (so skipped in the class).
- See Hahn, Todd and van der Klaauw (2001) [26], Dong (2018) [21], and Hansen (2022, Sections 21.10-11) [27] for details.

Estimation

Identification Results (re)

- Recall that the causal parameters τ_{SRD} and τ_{FRD} are identified respectively by (3) and (4):

$$\tau_{\text{SRD}} = \lim_{x \downarrow c} \mathbb{E}[Y_i | X_i = x] - \lim_{x \uparrow c} \mathbb{E}[Y_i | X_i = x],$$
$$\tau_{\text{FRD}} = \frac{\lim_{x \downarrow c} \mathbb{E}[Y_i | X_i = x] - \lim_{x \uparrow c} \mathbb{E}[Y_i | X_i = x]}{\lim_{x \downarrow c} \mathbb{E}[D_i | X_i = x] - \lim_{x \uparrow c} \mathbb{E}[D_i | X_i = x]}.$$

- For expositional purpose, denote one of Y_i and D_i by A_i , and define the following one-sided limits:

$$\mu_A^+ \equiv \lim_{x \downarrow c} \mathbb{E}[A_i | X_i = x], \quad \mu_A^- \equiv \lim_{x \uparrow c} \mathbb{E}[A_i | X_i = x].$$

- For estimating τ_{SRD} and τ_{FRD} , it suffices to estimate $\mu_Y^+, \mu_Y^-, \mu_D^+$, and μ_D^- .

Estimation Procedures: Local Polynomial Regressions

- The quantities μ_Y^+ , μ_Y^- , μ_D^+ , and μ_D^- are commonly estimated by local polynomial regressions (LPRs).
- As we studied before, the local constant (Nadaraya-Watson) regression causes the boundary bias. LPRs can circumvent such boundary bias (Hahn, Todd and van der Klaauw, 2001 [26]; Porter, 2003 [43]).
- A recommended choice of local polynomial order p is 1 or 2.¹
- On the other hand, there are several recommendations on bandwidth selection, which we study later.

¹Gelman and Imbens (2019) [24] argue that controlling for global high-order polynomials in regression discontinuity analysis is a flawed approach with three major problems: it leads to noisy estimates, sensitivity to the degree of the polynomial, and poor coverage of confidence intervals.

- Consider the following p -th order LPRs:

$$\hat{\beta}_A^+ \equiv \arg \min_{\beta \in \mathbb{R}^{p+1}} \sum_{i=1}^n 1(X_i \geq c) (A_i - [r_p(X_i - c)]^T \beta)^2 K\left(\frac{X_i - c}{h}\right),$$

$$\hat{\beta}_A^- \equiv \arg \min_{\beta \in \mathbb{R}^{p+1}} \sum_{i=1}^n 1(X_i < c) (A_i - [r_p(X_i - c)]^T \beta)^2 K\left(\frac{X_i - c}{h}\right),$$

where $p \geq 1$, $r_p(x) \equiv (1, x, \dots, x^p)$ is a vector of polynomials, $h > 0$ is bandwidth, and $K(\cdot)$ is a kernel function.

- The LPR estimators of μ_A^+ and μ_A^- are the first elements of $\hat{\beta}_A^+$ and $\hat{\beta}_A^-$, respectively:

$$\hat{\mu}_A^+ \equiv e_1^T \hat{\beta}_A^+, \quad \hat{\mu}_A^- \equiv e_1^T \hat{\beta}_A^-,$$

where $e_1 \equiv (1, 0, \dots, 0)^T$ is the first unit vector.

- The causal parameters τ_{SRD} and τ_{FRD} can be estimated respectively by

$$\hat{\tau}_{\text{SRD}} = \hat{\mu}_Y^+ - \hat{\mu}_Y^-, \quad \hat{\tau}_{\text{FRD}} = \frac{\hat{\mu}_Y^+ - \hat{\mu}_Y^-}{\hat{\mu}_D^+ - \hat{\mu}_D^-}.$$

Bandwidth Selection

- Under certain regularity conditions, we can show that

$$\mathbb{E}[\hat{\tau}_{\text{SRD}} - \tau_{\text{SRD}} | X_1, \dots, X_n] = h^{p+1} \mathcal{B},$$

$$\text{Var}[\hat{\tau}_{\text{SRD}} | X_1, \dots, X_n] = \frac{1}{nh} \mathcal{V},$$

$$\sqrt{nh} (\hat{\tau}_{\text{SRD}} - \tau_{\text{SRD}} - h^{p+1} \mathcal{B}) \xrightarrow{d} \text{Normal}(0, \mathcal{V}).$$

- The AMSE is given by

$$\text{AMSE}(\hat{\tau}_{\text{SRD}}) = h^{2(p+1)} \mathcal{B}^2 + \frac{1}{nh} \mathcal{V}.$$

- Imbens and Kalyanaraman (2012) [29] propose the (A)MSE optimal bandwidth, which satisfies that

$$h_{\text{IK2012}} \propto n^{-\frac{1}{2p+3}}.$$

Inference

- If we are interested in the point estimations of μ_Y^+ and μ_Y^- , it is sufficient to select IK2012's MSE optimal bandwidth.
- In practice, however, we are also interested in inference: IK2012's bandwidth does not satisfy that $nh^{2p+3} \rightarrow 0$. Then, we cannot eliminate the asymptotic bias.

How to select bandwidth?

1. Undersmoothing
2. Robust bias-corrected inference (CCT2014 [9], CCF2020 [8])
3. Uniformly honest inference (KR2018 [34])

- A naive solution to mitigate asymptotic bias is to use undersmoothing.
- By using undersmoothing bandwidth such that

$$\sqrt{nh}h^{p+1} \rightarrow 0,$$

the standard t statistic satisfies

$$\frac{\sqrt{nh}(\hat{\tau}_{\text{SRD}} - \tau_{\text{SRD}})}{\sqrt{\mathcal{V}_{\text{SRD}}}} \xrightarrow{d} \text{Normal}(0, 1).$$

- Calonico, Cattaneo and Titiunik (2014) [9], and Calonico, Cattaneo and Farrell (2020) [8] propose to correct asymptotic bias for valid statistical inference, not to eliminate the bias by undersmoothing.
- Let $\hat{\mathcal{B}}$ denote the estimator of \mathcal{B} based on LPRs using bandwidth b , which can differ from h . The bias-corrected estimator of τ_{SRD} is given by

$$\hat{\tau}_{\text{SRD}}^{\text{bc}} \equiv \hat{\tau}_{\text{SRD}} - h^{p+1} \hat{\mathcal{B}}.$$

- Under certain conditions on h and b and regularity conditions, CCT2014 [9] show that the robust bias-corrected t statistic satisfies

$$T_{\text{SRD}}^{\text{bc}} \equiv \frac{\sqrt{nh} (\hat{\tau}_{\text{SRD}}^{\text{bc}} - \tau_{\text{SRD}})}{\sqrt{\mathcal{V}_{\text{SRD}}^{\text{bc}}}} \xrightarrow{d} \text{Normal}(0, 1),$$

where $\mathcal{V}_{\text{SRD}}^{\text{bc}} \equiv \mathcal{V}_{\text{SRD}} + \mathcal{C}_{\text{SRD}}^{\text{bc}}$ and $\mathcal{C}_{\text{SRD}}^{\text{bc}}$ is a correction term.

- Using an estimator of $\mathcal{V}_{\text{SRD}}^{\text{bc}}$, the $1 - \alpha$ confidence interval of τ_{SRD} based on $T_{\text{SRD}}^{\text{bc}}$ is given by

$$\left[\hat{\tau}_{\text{SRD}}^{\text{bc}} - z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{\mathcal{V}}_{\text{SRD}}^{\text{bc}}}{nh}}, \hat{\tau}_{\text{SRD}}^{\text{bc}} + z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{\mathcal{V}}_{\text{SRD}}^{\text{bc}}}{nh}} \right]. \quad (5)$$

- CCF2020 [8] propose to select a bandwidth which minimizes the coverage rate error of (5).

Uniformly Honest Inference (KR2018)

- Kolesar and Rothe (2018) [34] propose another bias correction.
- Assuming that the true regression function is Lipschitz continuous, and analyzing the upper bound of the bias, they construct a confidence interval uniformly robust to any DGPs.
- Some characteristics of KR2018 are that they do not require $h \rightarrow 0$, and thus they allow the running variable to be discretely distributed.

- Consider the following LLR with the uniform kernel:

$$\min_{\alpha, \tau_h, \beta, \gamma} \sum_{i=1}^n 1(|X_i - c| \leq h) [Y_i - \alpha - \tau_h D_i - \beta(X_i - c) - \gamma D_i(X_i - c)]^2.$$

- Define

$$n_h = \sum_{i=1}^n 1(|X_i - c| \leq h),$$

$$\tilde{\tau}_h = \mathbb{E}[\hat{\tau}_h | X_1, \dots, X_n].$$

- Let $\frac{\hat{\sigma}^2}{n_h}$ be the estimator of $\text{Var}(\hat{\tau}_h | X_1, \dots, X_n)$.

- By simple calculation, we obtain

$$\frac{\sqrt{n_h}(\hat{\tau}_h - \tau_{\text{SRD}})}{\hat{\sigma}} = \frac{\sqrt{n_h}(\hat{\tau}_h - \tilde{\tau}_{\text{SRD}})}{\hat{\sigma}} + \frac{\sqrt{n_h}(\tilde{\tau}_h - \tau_{\text{SRD}})}{\hat{\sigma}}$$

- For any h (even if h violates $h \rightarrow 0$), the first term of RHS satisfies that

$$\frac{\sqrt{n_h}(\hat{\tau}_h - \tilde{\tau}_{\text{SRD}})}{\hat{\sigma}} \xrightarrow{d} \text{Normal}(0, 1).$$

- The second term is the bias term. Let us analyze its upper bound.

- Assume that, for a known constant K ,²

$$\mu(x) = \mathbb{E}[Y_i | X_i = x] \in \mathcal{M},$$

where

$$\mathcal{M} = \{\mu : |\mu'(a) - \mu'(b)| \leq K|a - b| \text{ for all } a, b < c \text{ and all } a, b > c\}.$$

- Then, for $\hat{\sigma}$ that is estimated with certain methods, we can obtain

$$\gamma_{sup} \equiv \sup_{\mu \in \mathcal{M}} \frac{\sqrt{n_h} |\tilde{\tau}_h - \tau_{\text{SRD}}|}{\hat{\sigma}}.$$

²We cannot choose K by data-driven procedures.

- Based on this result, we can construct the following confidence interval:

$$\text{CI} = \left[\hat{\tau}_h - \text{cv}_{1-\alpha}(\gamma_{sup}) \frac{\hat{\sigma}}{\sqrt{n_h}}, \hat{\tau}_h + \text{cv}_{1-\alpha}(\gamma_{sup}) \frac{\hat{\sigma}}{\sqrt{n_h}} \right],$$

where $\text{cv}_{1-\alpha}(\gamma)$ denotes the $1 - \alpha$ quantile of $|\text{Normal}(\gamma, 1)|$.

- The CI above is an asymptotically uniform confidence interval with respect to \mathcal{M} :

$$\lim_{n \rightarrow \infty} \inf_{\mu \in \mathcal{M}} \mathbb{P}_{\mu}(\tau_{\text{SRD}} \in \text{CI}) \geq 1 - \alpha,$$

which KR2018 [34] call the **honest** confidence interval.

Covariates

Purpose of Covariate Adjustment

- Efficiency gains in the RD effect estimator
- Not for relaxing identification assumptions (i.e., not for the unconfoundedness)

RD with Covariates

- Armstrong and Kolesar (2018) [5]
- Calonico, Cattaneo, Farrell and Titiunik (2019) [10]
- Frolich and Huber (2019) [22]

RD with High-Dimensional Covariates

- Kreiss and Rothe (2023) [35]
- Arai, Otsu and Seo (2024) [4]
- Noack, Olma and Rothe (2025) [40]
- Review: Chernozhukov et al. (2025, Chapter 17) [17]

Covariate Adjustment Relying on Robinson (1988)

- Relying on Robinson's (1988) [45] estimation of semiparametric partially linear model, we can adjust covariates to RD estimation.
- The estimator reaches the semiparametric efficiency bound. However, the finite-sample property is not so good.
- See Section 21.7 of Hansen (2022) [27] for details.

Covariate-Adjusted RD Estimation (CCFT2019)

- Calonico, Cattaneo, Farrell and Titiunik (2019) [10] recommend to implement the WLS based on

$$\tilde{Y}_i = \tilde{\alpha} + D_i \tilde{\tau} + X_i \tilde{\beta}_- + D_i X_i \tilde{\beta}_+ + Z_i^T \tilde{\gamma},$$

where

- $c = 0$ is a normalized cutoff point,
- Z_i is a vector of auxiliary covariates of units satisfying $X_i \in \{-h, h\}$,
- the weight function is assumed to be the uniform or triangular kernel.

That is, they consider a LLR estimator based on a kernel with bounded support.

- Under the assumption that there are no treatment effects on the covariates, $\tilde{\tau}$ is a consistent estimator of τ_{SRD} .
- Inference procedures based on the estimator $\tilde{\tau}$ are similar to the ones discussed in CCT2014 [9] and CCF2020 [8].

How to select a small subset of covariates for efficiency improvement?

- Repeatedly, it is not necessary to address issues of selection bias or to implement a double-selection-type method.
- Kreiss and Rothe (2023) [35] : a “localized” version of Lasso regression
 - Related literature that consider the localized lasso include Su et al. (2019) [49]. They handle high dimensional covariates in a nonparametric setup, namely a continuous treatment model.
- Arai, Otsu and Seo (2024) [4] : the debiased Lasso (Zhang and Zhang, 2014 [52])
- Noack, Olma and Rothe (2025) [40] : cross-fitting

Falsification Test

Identification Assumptions

Identification Assumption (2) in Sharp RD

For each $d \in \{0, 1\}$,

$\mathbb{E}[Y_i(d)|X_i = x]$ is continuous at $x = c$.

- **Counterfactual:** The continuity of $\mathbb{E}[Y_i(d)|X_i = x]$ at $x = c$ cannot be directly examined, since $Y_i(0)$ and $Y_i(1)$ are unobservable under $X_i \geq c$ and $X_i < c$, respectively.
- Instead, researchers often examine certain necessary conditions.
- **No manipulation:** Under the above assumptions, it is necessary for the density of running variable X_i to be continuous at the cutoff point.

- Testing the continuity of the density of the assignment variable:
 - McCrary (2008) [38]
 - Otsu, Xu and Matsushita (2013) [42]
 - Cattaneo, Jansson and Ma (2020) [15]
 - Software packages by CJM2020: `lpdensity`, `rddensity`
 - Imai and Okamoto (2024) [28]
- Testing the continuity of the conditional distribution of covariates:
 - Lee (2008) [36]
 - Canay and Kamat (2018) [11]
 - Fusejima, Ishihara and Sawada (2025) [23]

Placebo Test

- Take several placebo cutoff points.
- At a placebo cutoff point $c' < c$, researchers can observe whether the density of untreated potential outcome $Y_i(0)$ is continuous or not.
- Similarly, at $c' > c$, they can observe whether the density of treated potential outcome $Y_i(1)$ is continuous or not.
- If the continuity is observed, then there arises some plausibility of the identification assumptions in Theorem 21.1.
- However, it is just a plausibility. Note that the continuity at placebo cutoff points is **neither necessary nor sufficient** for the identification assumptions.

- Recent works argue that, in general, researchers should not implement such pretesting.
 - Roth (2022) [46] : Pre-trend test
 - Sueishi (2023) [50] : Hausman test
- Pretesting analysis in the RD setting can be found in Section 5.2 of Fusejima, Ishihara and Sawada (2025) [23].

Practical Recommendation

Practical Recommendation in Sharp RD

- The polynomial order p of LPRs should be 1 or 2.
- For point estimation, use IK2012's (A)MSE optimal bandwidth.
- For inference, use CCT2014 & CCF2020's bandwidth.
- The common package `rdrobust` is equipped with point estimation relying on IK2012 and robust bias-corrected inference by CCT2014 & CCF2020.
- Observable covariates can be utilized to gain efficiency of the sharp RD estimator.
- Note that pretesting for Assumption (2) may distort the estimation and inference results.

Empirical Application

“Waiting for Life”

- The first study relying on RD: Thistlethwaite and Cambell (1960) [51]
- Angrist and Lavy (1999) [3], Black (1999) [7]
 - RD was “waiting for life” (Cook, 2008 [18])
- Hahn, Todd, and Klaauw (2001) [26] formalize general RDDs and establish identification results for treatment effects.

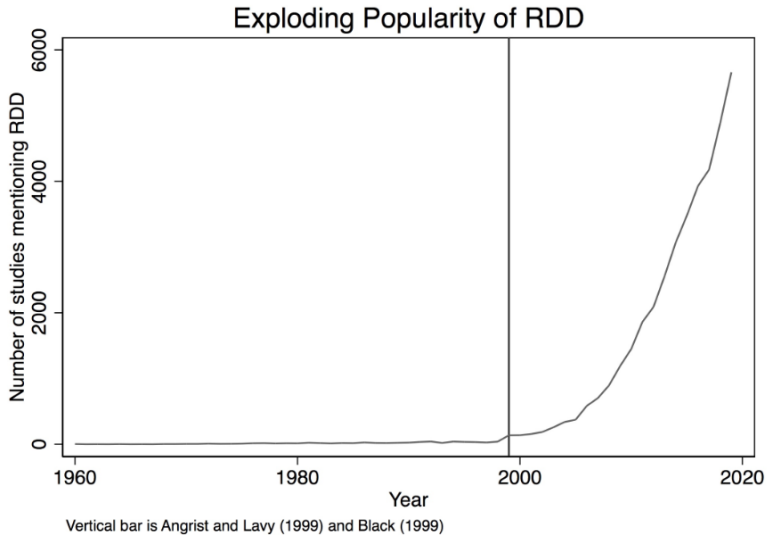


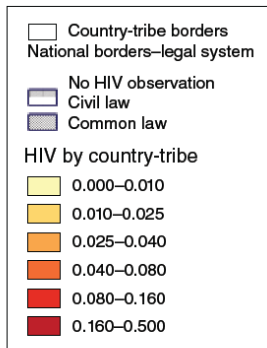
Figure 6.1 of Cunningham (2021) [19]

Recent Empirical RD Studies

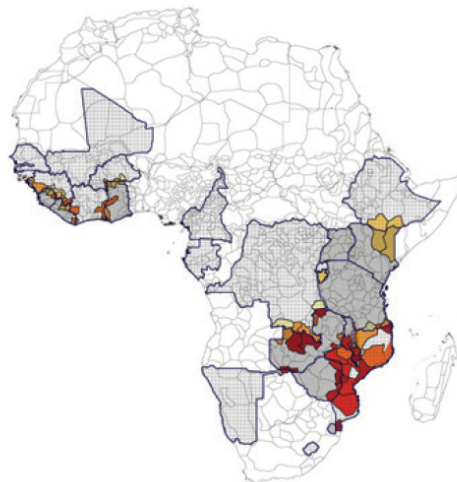
- Here we review Anderson (2018) [2] and Kawai et al. (2023) [32].
 - For more details than below, see the OG papers and auxiliary review slides by me.
 - Slides for Anderson (2018)
 - Slides for Kawai et al. (2023)
- Other empirical RD studies:
 - Ludwig and Miller (2007) [37], Lee (2008) [36], Matsudaira (2008) [39], Battistin et al. (2009) [6], Carpenter, Christopher and Carlos Dobkin (2009) [12], Greenstone, Hornbeck and Moretti (2010) [25], Abdulkadiroglu, Angrist and Pathak (2014) [1], Ito (2014) [30], Kleven et al. (2014) [33], Shigeoka (2014) [47], Shigeoka (2016) [48], Ito and Sallee (2018) [31], Oizer (2018) [41], etc.

- Anderson (2018) [2] examines causal relationship between legal systems and female HIV infection rates in sub-Saharan Africa.
- RD, motivated by Dell (2010) [20]:
 - **As-if random borders** can mitigate an endogeneity that emerges within ethnicities.
- Result 1 (HIV positive rates)
 - Female: common law countries $>$ civil law countries
 - Male: no significant difference
- Result 2 (Contraception use)
 - Female: common law countries $<$ civil law countries
 - Male: common law countries $<$ civil law countries
- Common Law \Rightarrow Female bargaining power \downarrow
 \Rightarrow Negotiation for safe sex practices $\times \Rightarrow$ HIV prevalence \uparrow

Split Ethnic Groups with Different Legal Origins



Panel C. HIV (women): split ethnic groups with different legal origins



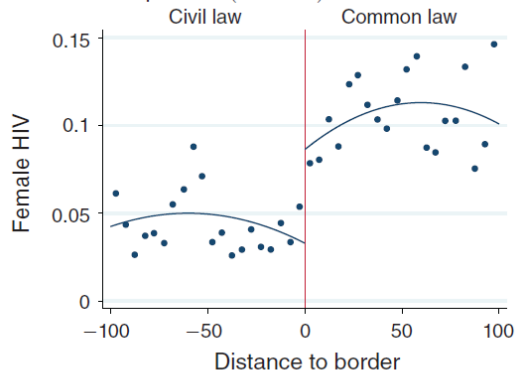
- Model:

$$Y_{rcepi} = \alpha_0 + \alpha_1 L_{rc} + \alpha_2 X_{rc} + \alpha_3 X_{rcep} + \alpha_4 X_{rcepi} + f(BD_{rcep}) \\ + \delta_e + \gamma_r + \lambda_t + \epsilon_{rcepi}$$

- Subscripts: **r**egion, **c**ountry, **e**thnic homeland, **p**ixel
- Y_{rcepi} : an outcome of interest
- L_{rc} : common law legal system indicator
- $X_{rc}, X_{rcep}, X_{rcepi}$: vectors of controls
- $f(BD_{rcep})$: a second-order RD polynomial of the distance from the centroid of pixel to the nearest national border with different legal origins
- δ_e, γ_r : fixed effects w.r.t. ethnicity and region, respectively
- ϵ_{rcepi} : clustered at the ethnicity and country level
- λ_t : years of survey

HIV Prevalence Rates

Panel A. HIV positive (*females*)



Panel B. HIV positive (*males*)

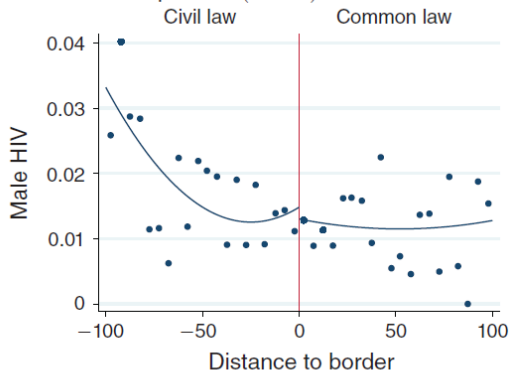


FIGURE 2. HIV POSITIVE

HIV Prevalence Rates

TABLE 1—HIV POSITIVE: FEMALES AGED 15–49

Variable	Whole sample			Non-Muslim Non-Polygynous	Muslim Polygynous
	≤ 200 km	≤ 150 km	≤ 100 km	≤ 100 km	≤ 100 km
Common law	0.016 (0.006)	0.018 (0.006)	0.019 (0.007)	0.016 (0.006)	0.007 (0.013)
Observations	118,903	99,511	77,336	55,507	21,829

Notes: Standard errors are clustered at the ethnic and country level using the approach of Cameron, Gelbach, and Miller (2011). All estimations include: country, individual, and pixel controls; region fixed effects; ethnic fixed effects; second-order RD polynomial of distance to national border; and the year of the survey. Refer to the online Appendix for details on the data.

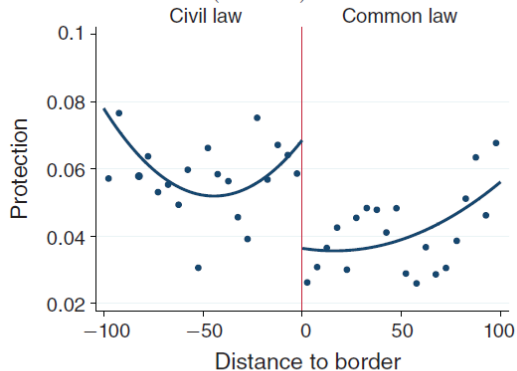
TABLE 2—HIV POSITIVE: MALES AGED 15–49

Variable	Whole sample			Non-Muslim Non-Polygynous	Muslim Polygynous
	≤ 200 km	≤ 150 km	≤ 100 km	≤ 100 km	≤ 100 km
Common law	0.001 (0.006)	0.001 (0.005)	–0.001 (0.005)	–0.003 (0.005)	0.002 (0.01)
Observations	50,754	40,780	31,189	24,261	6,928

Notes: Standard errors are clustered at the ethnic and country level using the approach of Cameron, Gelbach, and Miller (2011). All estimations include country, individual, and pixel controls; region fixed effects; ethnic fixed effects; second-order RD polynomial of distance to national border; and the year of the survey. Refer to the online Appendix for details on the data.

Protective Contraception

Panel A. Protection (*females*)



Panel B. Protection (*males*)

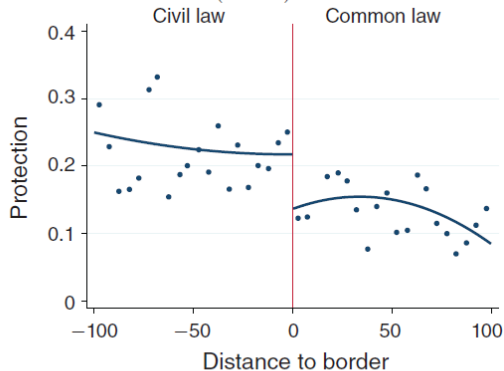


FIGURE 3. PROTECTION

Protective Contraception

TABLE 3—PROTECTIVE CONTRACEPTION: FEMALES AGED 15–49

Variable	Whole sample			Non-Muslim Non-Polygynous	Muslim Polygynous
	≤ 200 km	≤ 150 km	≤ 100 km	≤ 100 km	≤ 100 km
Common law	–0.018 (0.006)	–0.019 (0.006)	–0.019 (0.007)	–0.024 (0.01)	–0.008 (0.007)
Observations	117,263	97,285	76,698	55,261	21,437

Notes: Standard errors are clustered at the ethnic and country level using the approach of Cameron, Gelbach, and Miller (2011). All estimations include country, individual, and pixel controls; region fixed effects; ethnic fixed effects; second-order RD polynomial of distance to national border; and the year of the survey. Refer to the online Appendix for details on the data.

TABLE 4—PROTECTIVE CONTRACEPTION: MALES AGED 15–49

Variable	Whole sample			Non-Muslim Non-Polygynous	Muslim Polygynous
	≤ 200 km	≤ 150 km	≤ 100 km	≤ 100 km	≤ 100 km
Common law	–0.07 (0.02)	–0.07 (0.02)	–0.07 (0.02)	–0.08 (0.02)	–0.003 (0.02)
Observations	81,873	67,887	52,902	46,016	6,886

Notes: Standard errors are clustered at the ethnic and country level using the approach of Cameron, Gelbach, and Miller (2011). All estimations include country, individual, and pixel controls; region fixed effects; ethnic fixed effects; second-order RD polynomial of distance to national border; and the year of the survey. Refer to the online Appendix for details on the data.

- Cartels participating in procurement auctions frequently use bid rotation or prioritize incumbents to allocate contracts.
- However, establishing a link between observed allocation patterns and firm conduct has been difficult: **Cartel? Cost-based competition?**
- Data-driven screens to flag suspicious firm conduct:
 - Under competition, differences in backlog or incumbency status between close winners and close losers should vanish.
 - Under cartels, bids are generated by collusive bidding. Then, the differences in these variables between close winners and close losers need not disappear.
 - Their tests of non-competitive behaviour seek to detect discontinuities in the distribution of economically relevant covariates around close winners and close losers.
- Empirical examples:
 - Ohio milk auctions (Porter and Zona, 1999 [44])
 - Auctions for construction projects let by municipalities in Tohoku, Japan

Collusion? Competition?

- Bidders with low levels of backlog (firms that have not won many auctions in the recent past) are more likely to win than bidders with high levels of backlog.
- **Difficulty** in discriminating between competitive and non-competitive bid rotation and incumbency patterns:
 - Suppose that firms' procurement costs are increasing with backlog.
 - Even if firms are competitive, on average, firms with lower backlog will have lower costs and be more likely to win an auction than firms with higher backlog.
 - In this environment, a test seeking to detect collusive bid rotation by comparing the unconditional backlog of winners and losers would yield false positives.

How to Detect?

- Kawai et al. (2023) [32] propose to compare the backlog of a selected group of firms: bidders that win or lose by a small margin.
- Under **competition**, no firm can consistently be a marginal winner or a marginal loser. Winning or losing should be as-if-random conditional on close bids. As a result, close winners and losers should be statistically similar.
- Under **collusion**, close winners have consistently lower levels of backlog than close losers, which is evidence of collusive bid rotation.

Regression Discontinuity Approach (based on CCT2014)

- Let $\Delta_{i,t} \equiv b_{i,t} - \wedge b_{-i,t}$ denote the difference between the bid of firm i , and the most competitive alternative bid at time t .
- If $\Delta_{i,t} < 0$, bidder i wins the auction; if $\Delta_{i,t} > 0$, bidder i loses.
- Let $x_{i,t}$ be a measure (observed by the econometrician) of firm i 's backlog before bidding at time t (alternatively it could be incumbency, or another relevant covariate).
- Define β the difference in average backlog between close losers and close winners:

$$\beta = \lim_{\epsilon \downarrow 0^+} \mathbb{E}[x_{i,t} | \Delta_{i,t} = \epsilon] - \lim_{\epsilon \uparrow 0^-} \mathbb{E}[x_{i,t} | \Delta_{i,t} = \epsilon] \quad (6)$$

- Test the null: $\mathbb{H}_0 : \beta = 0$.
 - When x denotes backlog, we expect β to be strictly positive under bid rotation.
 - When x denotes incumbency status, we expect β to be strictly negative if the cartel allocates market shares according to incumbency.
- Reject $\mathbb{H}_0 \implies$ Reject “competition” (some evidence of collusion)

Japanese Procurement Auction (Tohoku Region)

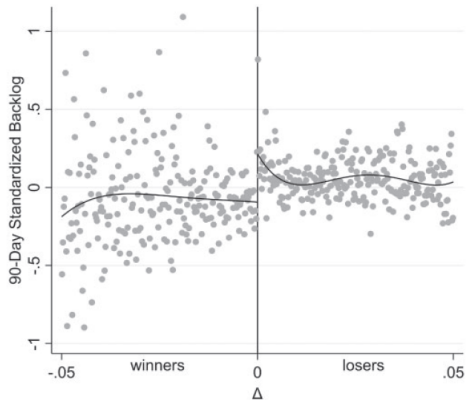








FIGURE 1




Binned scatter plot of standardized backlog, Japanese municipal auctions.




Notes: For each firm i and auction t , the standardized backlog of firm i at t is the Yen denominated amount of work it won in the 90 days prior to auction t , re-expressed in units of standard deviation from the firm's time-series average. The figure is a binned scatter plot of this measure against $\Delta_{i,t}$. See Section 5 for details.





References






-  Abdulkadiroglu, Atila; Angrist, Joshua D.; Pathak, Parag A. (2014). The Elite Illusion: Achievement Effects at Boston and New York Exam Schools. *Econometrica*, 82(1):137-196.
-  Anderson, S. Legal origins and female HIV. *American Economic Review*, 108(6):1407-1439.
-  Angrist, Joshua D.; Lavy, Victor. (1999). Using Maimonides ' Rule to Estimate the Effect of Class Size on Scholastic Achievement. *Quarterly Journal of Economics*, 114(2):533-575
-  Arai, Yoichi; Otsu, Taisuke; Seo, Myung Hwan. (2024). Regression Discontinuity Design with Potentially Many Covariates. *arXiv preprint arXiv:2109.08351*




-  Armstrong, T.B. and M. Kolesar. (2018). Optimal inference in a class of regression models. *Econometrica*, 86, 655-683.
-  Battistin, Erich; Brugiavini, Agar; Rettore, Enrico; Weber, Guglielmo. (2009). The Retirement Consumption Puzzle: Evidence from a Regression Discontinuity Approach. *American Economic Review*, 99(5):2209-2226.
-  Black, Sandra E. (1999). “ Do Better Schools Matter? Parental Valuation of Elementary Education. ” *Quarterly Journal of Economics*, 114(2):577-599.
-  Calonico, S., M. D. Cattaneo and M. H. Farrell (2020). Optimal bandwidth choice for robust bias-corrected inference in regression discontinuity designs. *The Econometrics Journal* 23(2), 192-210.




-  Calonico, S., M. D. Cattaneo, and R. Titiunik (2014). Robust nonparametric confidence intervals for regression-discontinuity designs. *Econometrica* 82(6), 2295-2326.
-  Calonico, Sebastian; Cattaneo, Matias D.; Farrell, Max H.; Titiunik, Rocio. (2019). Regression Discontinuity Designs Using Covariates. *The Review of Economics and Statistics*, 101(3):442-451
-  Canay, I. A. and V. Kamat (2018). Approximate permutation tests and induced order statistics in the regression discontinuity design. *Review of Economic Studies* 85, 1577-1608.




-  Carpenter, Christopher; Dobkin, Carlos. The Effect of Alcohol Consumption on Mortality: Regression Discontinuity Evidence from the Minimum Drinking Age. *American Economic Journal: Applied Economics*, 1(1):164-182.
-  Cattaneo, Matias D., Nicolas Idrobo, and Rocio Titiunik. (2019). *A Practical Introduction to Regression Discontinuity Designs: Foundations*. Elements in Quantitative and Computational Methods for the Social Sciences. Cambridge University Press.
-  Cattaneo, Matias D., Nicolas Idrobo, and Rocio Titiunik. (2024). *A Practical Introduction to Regression Discontinuity Designs: Extensions*. Elements in Quantitative and Computational Methods for the Social Sciences. Cambridge University Press.





-  Cattaneo, M. D., M. Jansson and X. Ma (2020). Simple local polynomial density estimators. *Journal of the American Statistical Association* 115(531), 1449-1455.
-  Cattaneo, Matias D. and Roberto Titiunik. “Regression Discontinuity Designs.” *Annual Review of Economics*(14), 821-851.
-  Chernozhukov, Victor; Hansen, Christian; Kallus, Nathan; Spindler, Martin; Syrgkanis, Vasilis. (2025). Applied Causal Inference: Machine Learning and AI. CausalML Book, Version 0.1.1. Available at: <https://causalml-book.org>.
-  Cook, Thomas D. (2008). Waiting for Life to Arrive ’ : A History of the Regression-Discontinuity Design in Psychology, Statistics and Economics. *Journal of Econometrics*, 142(2):636-654.





-  Cunningham, Scott. (2021). *Causal Inference: The Mixtape*. Yale University Press
-  Dell, Melissa. (2010). The Persistent Effects of Peru's Mining Mita. *Econometrica*, 78(6):1863-1903.
-  Dong, Yingying. (2018). Alternative Assumptions to Identify LATE in Fuzzy Regression Discontinuity Designs. *Oxford Bulletin of Economics and Statistics*, 80(5):1020-1027.
-  Frolich, M. and M. Huber. (2019). Including covariates in the regression discontinuity design. *Journal of Business and Economic Statistics*, 37, 736-748.
-  Fusejima, K., T. Ishihara and M. Sawada (2025). A unified diagnostic test for regression discontinuity designs. arXiv:2205.04345v4.






-  Gelman, A.; Imbens, G.W. Why high-order polynomials should not be used in regression discontinuity designs. (2019). *Journal of Business & Economic Statistics*, 37(3):447-456.
-  Greenstone, Michael; Hornbeck, Richard; Moretti, Enrico. (2010). Identifying Agglomeration Spillovers: Evidence from Winners and Losers of Large Plant Openings. *Journal of Political Economy*, 118(3):536-598.
-  Hahn, Jinyong; Todd, Petra; van der Klaauw, Wilbert. (2001) Identification and Estimation of Treatment Effects with a Regression-Discontinuity Design. *Econometrica*, 69(1):201-209.
-  Hansen, B. E. (2022). *Econometrics*. Princeton.





-  Imai, Shunsuke; Okamoto, Yuta. (2024). Kernel Choice Matters for Boundary Inference Using Local Polynomial Density: With Application to Manipulation Testing. *arXiv preprint arXiv:2306.07619*
-  Imbens, G. W. and K. Kalyanaraman (2012). Optimal bandwidth choice for the regression discontinuity estimator. *The Review of Economic Studies* 79(3), 933-959.
-  Ito, Koichiro. (2014). Do Consumers Respond to Marginal or Average Price? Evidence from Nonlinear Electricity Pricing. *American Economic Review*, 104(2):537-563.






-  Ito, Koichiro; Sallee, James M. (2018). The Economics of Attribute-Based Regulation: Theory and Evidence from Fuel-Economy Standards. *Review of Economics and Statistics*, 100(2):319-336.
-  Kawai, Kei; Nakabayashi, Jun; Ortner, Juan; Chassang, Sylvain. (2023). Using Bid Rotation and Incumbency to Detect Collusion: A Regression Discontinuity Approach. *The Review of Economic Studies*, 90(1):376-403.
-  Kleven, Henrik J.; Landais, Camille; Saez, Emmanuel; Schultz, Esben A. Migration and Wage Effects of Taxing Top Earners: Evidence From the Foreigners ' Tax Scheme in Denmark. *Quarterly Journal of Economics*, 129(1):333-378.

-  Kolesar, Michal; Rothe, Christoph. (2018). Inference in Regression Discontinuity Designs with a Discrete Running Variable. *American Economic Review*, 108(8):2277-2304
-  Kreiss, Alexander; Rothe, Christoph. (2023). Inference in Regression Discontinuity Designs with High-dimensional Covariates. *The Econometrics Journal*, 26(2):105-123.
-  Lee, D. S. (2008). Randomized experiments from non-random selection in US House elections. *Journal of Econometrics* 142(2), 675-697.
-  Ludwig, Jens; Miller, Douglas L. (2007). Does Head Start Improve Children ' s Life Chances? Evidence from a Regression Discontinuity Design. *Quarterly Journal of Economics*, 122(1):159-208.

-  McCrary (2008). Manipulation of the running variable in the regression discontinuity design: A density test. *Journal of Econometrics* 142(2), 698-714.
-  Matsudaira, Jordan D. (2008). Mandatory summer school and student achievement. *Journal of Econometrics*, 142(2):829-850.
-  Noack, C.; Olma, T.; Rothe, C. (2025). Flexible Covariate Adjustments in Regression Discontinuity Designs. *arXiv preprint arXiv:2107.07942*
-  Ozier, Owen. (2018). Exploiting Externalities to Estimate the Long-Term Effects of Early Childhood Deworming. *American Economic Journal: Applied Economics*, 10(3):235-262.

-  Otsu, T., K. L. Xu, and Y. Matsushita (2013). Estimation and inference of discontinuity in density. *Journal of Business and Economic Statistics* 31(4), 507-524.
-  Porter, Jack (2003). Estimation in the regression discontinuity model. *Unpublished Manuscript, Department of Economics, University of Wisconsin at Madison*, 5-19.
-  Porter, R. H., & Zona, J.D. (1999). Ohio school milk markets: An analysis of bidding. *The RAND Journal of Economics*, 30(2), 263-288.
-  Robinson, Peter M. (1988). Root-N-Consistent Semiparametric Regression. *Econometrica*, 56(4):931-954
-  Roth, J. (2022). Pretest with caution: Event-study estimates after testing for parallel trends. *American Economic Review: Insights* 4(3), 305-322.

-  Shigeoka, Hitoshi. (2014). The Effect of Patient Cost Sharing on Utilization, Health, and Risk Protection. *American Economic Review*, 104(7):2152-2184.
-  Shigeoka, Hitoshi. (2015). School Entry Cutoff Date and the Timing of Births. NBER Working Paper No. 21402. Available at:
<https://www.nber.org/papers/w21402.pdf>.
-  Su, L., T. Ura, and Y. Zhang. (2019). Non-separable models with high-dimensional data. *Journal of Econometrics*, 212, 646-677.
-  Sueishi, N. (2023). A misuse of specification tests. arXiv:2211.11915v1.

-  Thistlethwaite, Donald, and Donald Campbell. (1960). Regression-Discontinuity Analysis: An Alternative to the Ex-Post Facto Experiment. *Journal of Educational Psychology* 51, 309-317.
-  Zhang, Cun-Hui; Zhang, Stephanie S. (2014) Confidence Intervals for Low-Dimensional Parameters in High-Dimensional Linear Models. *Journal of the Royal Statistical Society. Series B (Statistical Methodology)*, 76(1):217-242.
-  川口康平, 澤田真行 (2024) 『因果推論の計量経済学』 日本評論社.
-  高野久紀 (2025) 『開発経済学：実証経済学へのいざない』 日本評論社.
-  末石直也 (2024) 『データ駆動型回帰分析』 日本評論社.